Foundations and Trusses

Overview: In these activities, participants use right triangles, similarity and the Pythagorean Theorem in applications involving laying a foundation for a house and building trusses for a roof.

Objective: **Mathematics TEKS**

(6.3.B) The student is expected to represent ratios and percents with concrete models, fractions and decimals.

(6.8.D) The student is expected to convert measures within the same measurement system (customary and metric) based on relationships between units.

(6.11.A, 7.13.A, 8.14.A)
The student is expected to identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics.

(7.3.B) The student is expected to estimate and find solutions to application problems involving proportional relationship such as similarity, scaling, unit costs, and related measurement units.

(7.6.D) The student is expected to use critical attributes to define similarity.

(7.8.C, 8.7.B) The student is expected to use geometric concepts and properties to solve problems in fields such as art and architecture.

(7.9) The student is expected to estimate measurements and solve application problems involving length (including perimeter and circumference), area, and volume.

(8.1.B) The student is expected to select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships.

(8.3.B) The student is expected to estimate and find solutions to application problems involving percents and proportional relationships such as similarity and rates.

(8.9.A) The student is expected to use the Pythagorean Theorem to solve real-life problems.

(8.9.B) The student is expected to use proportional relationships in similar shapes to find missing measurements.

(G.f.2) The student uses ratios to solve problems involving similar figures.

(G.f.3) In a variety of ways, the student develops, applies, and justifies triangle similarity relationships, such as right triangle ratios, trigonometric ratios and Pythagorean triples.

Mathematics TAKS Grades 6 – 8

Objective 1: The student will demonstrate an understanding of numbers, operations, and quantitative reasoning. (8.1B)

Objective 2: The student will demonstrate an understanding of patterns relationships, and algebraic reasoning. (6.3B), 7.3B, 8.3B)

Objective 3: The student will demonstrate an understanding of geometry and spatial reasoning. (7.6.D)

Objective 4: The student will demonstrate an understanding of the concepts and uses of measurement. (6.8D, 7.8C, 7.9, 8.7B, 8.9A, 8.9B)
Objective 6: The student will demonstrate an understanding of the mathematical processes and tools used in problem solving. (6.13.A, 7.15.A, 8.16.A)
Objective 8: The student will demonstrate an understanding of the concepts and uses of measurement and similarity.

Terms: 3-4-5 right triangle, diagonals, Pythagorean triple, rigidity, pitch of roof, span of roof, altitude, similarity, perpendicular, reflection

Materials: **Activity 1:** hammers, measuring tape, stakes, string, tape measures, scissors
Activity 2: 7 4 inch straws, 2 long straws, string, scissors, grid paper, ruler, calculator, dynamic geometry software with truss script (optional)

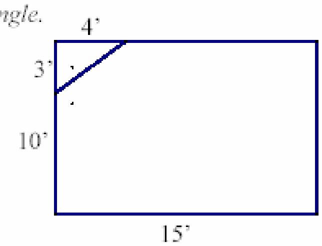
Procedures: *Put participants in groups of 4-5 so that each group has a balance of different grade level teachers.*

Activity 1: Laying out a Foundation

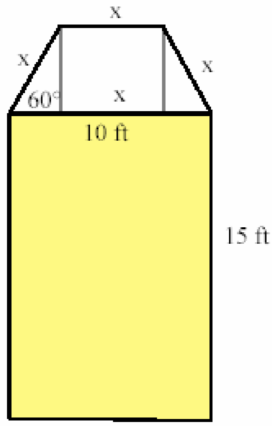
Show the transparency of activity 1 and discuss the content.

1. **Using the supplies you have been given, stake out a 10' x 15' rectangle and use string to outline it. Use the 3-4-5 Pythagorean triple to adjust the angles at the vertices of the rectangle so that you are sure that they are 90°.**
Participants are asked to use their supplies to stake out a 10' x 15' rectangle outdoors. After staking out the first side of the rectangle, they must make an estimate where the adjacent side will be and lay out the estimated location. They should then use the 3-4-5 Pythagorean Triple to adjust the angle of that adjacent side until the triple is established, confirming that the angle is a right angle. They will then stake the next side, following the same process, and finally, they should connect the fourth side. Participants need to make sure the opposites sides of the quadrilateral remain 10 feet and 15 feet as they make their adjustments.
If it is not possible to go outside because of rain or the physical situation, the activity can be done in any large room by taping the string to the floor. The dimensions can be sized down, but try to stake out as large a rectangle as possible. The larger the rectangle is the more dramatic the activity.

2. **Check your rectangle to be sure it is accurate. What other method can you use to ensure that the angles of your rectangle are right angles?**
Have participants check their rectangles for accuracy. They can check that the diagonals are congruent, for example, or use other Pythagorean triples such as 6,8,10 or 5, 12, 13. Discuss why the 3,4,5 triple is easy to use and whether there are other ways of confirming the accuracy of the rectangle.



3. Next, we would like to add a bay window to one of the 10 foot sides of the foundation. Using the string and stakes, lay out the foundation for the bay window so that the trapezoid formed by the three walls of the bay window and the 10 foot side of the rectangle is half of a regular hexagon. Find the lengths of the three sides of the bay window and describe the method used to lay out this part of the foundation.
The sides of the bay window can be found using 30-60-90 triangle ratios.



10=x/2 + x/2 + x
10=2x
So, x=5 ft.

One method participants can use to lay this part of the foundation is described below. Mark a length of 5 feet from one vertex on the 10 foot side of the rectangle. Cut two strings of length five. Anchor one of the 5 foot lengths at the vertex of the rectangle and the other at the 5 foot mark. Swing each of these out until they meet. Place a stake at this point. This creates an equilateral triangle with side lengths 5 feet and a 60° angle. Repeat this from the other vertex.

Extension: Check other angles around the building to determine if they are right angles: doors, windows, corners of rooms, and angles of walls to floor. Use another Pythagorean Triple to recheck the angles.
As an extension, participants check the angles of the corners, walls, windows and doors in the classroom and try other Pythagorean triples to confirm that the angles measured are right angles. In reality, many corners we assume are right angles are not very exact. Anyone who has ever tried to hang window coverings may have discovered this in his own home.

Activity 2: Building Trusses

Triangles are the simplest polygons and are the most useful polygons. This activity will examine how triangles are used in building roof trusses.

Show the transparencies of structures and discuss what geometric shapes are common in each of the structures shown. Triangles are used in the frame or trusses.

Ask: Why do you think triangles are used in such structures? Strong, rigid, relatively lightweight

Ask: Where else do you see triangles used for support? Bridges, towers, roof trusses, playground equipment, shelf bracing, bicycle frames, guy wires on telephone poles ...

Ask: What does it mean for a figure to be rigid? The figure cannot be distorted under stress.

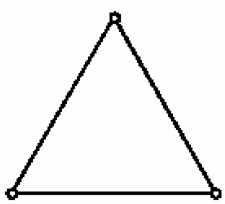
1. **Thread the string through three of the 4-inch straws and tie a knot to create a triangle. Repeat this procedure with four of the 4-inch straws to create a quadrilateral. Compare and contrast the rigidity of each figure. If a figure is not rigid, how can you make it rigid?**

   Give each group the materials listed for Activity 2 and have the groups create a triangle and quadrilateral with the 4-inch straws. Model this step with participants and place the straw figures on the overhead.

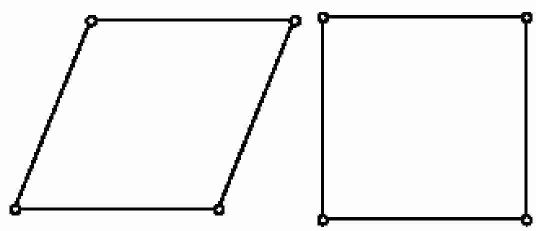
   Ask: What kind of triangle did you create? (Equilateral triangle)

   Ask: What kind of quadrilateral did you create? (Rhombus)

   If participants respond that they created a square ask if the quadrilateral will always remain a square. If not, what is a more general name for a quadrilateral with 4 equal sides?



rigid



not rigid

Notice in the figure above, the rhombus can be distorted into other rhombi – even a square, but the triangle cannot be distorted. So, the triangle is rigid and the quadrilateral is not.

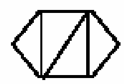
Ask: Would the triangle still be rigid if the straws were different lengths? Yes

To make the quadrilateral rigid, cut a straw the length of the diagonal of the quadrilateral you want to stabilize.

- Calculate the length of the diagonal. $4\sqrt{2}\ in \approx 5.656\ in$
- What kind of special right triangle is formed by the sides of the square and the diagonal? 45°-45°-90°
- How does the calculated length of the diagonal compare to the cut length of the straw?

Thread string through the straw and tie it to opposite vertices of the quadrilateral. In doing this, you create two triangles which are rigid. Some groups may want to add two diagonals. Demonstrate on the overhead that this is not necessary.

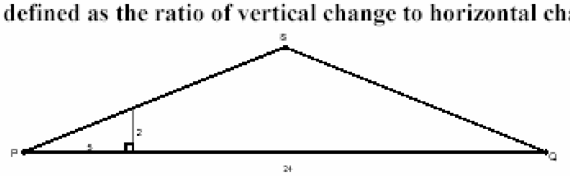
2. **How could you make a pentagon rigid? A hexagon? What is the minimum number of diagonals needed to make an n-gon rigid?**
   To make a pentagon rigid, add two diagonals. To make a hexagon rigid, add three diagonals. To make an n-gon rigid add (n-3) diagonals which creates (n-2) triangles.
   - Do the diagonals have to come from a common vertex?
     No



Note: A truss is a rigid framework used in bridges and roofs. All trusses rely on triangles to ensure rigidity. As a refinement of solid beams, trusses are an efficient means to span long distances with a minimum of materials. Made up of smaller elements connected with hinged joints, trusses take advantage of the concept of triangular rigidity.

3. **Below is a diagram of a roof with a $\frac{2}{5}$ pitch spanning 24 feet. (PS=QS)**

   The pitch of a roof is defined as the ratio of vertical change to horizontal change.



   a. **Calculate the measure of angle SPQ.**
      $\tan^{-1}(.4) \approx 21.8°$
   b. **Draw the altitude from point S to $\overline{PQ}$ and label the intersection R. Find SR.**
      The altitude to the base of an isosceles triangle intersects the base at its midpoint. So, PR=RQ=12 ft. To find the length of the altitude, a, from S using similar triangles, we see that $\frac{2}{5} = \frac{a}{12}$, so a = 4.8 ft = 4 ft 9.6 in.
   c. **Find PS. Express your answer in terms of feet and inches.**
      Now, we can use the Pythagorean theorem to find PS.
      $12^2 + 4.8^2 = PS^2$, so $PS \approx 12.9$ ft ≈ 12 ft 10.8 in
      What mathematical term is the pitch of the roof referring to? Slope. Note: some teachers may say that it is referring to the tangent of the angle of elevation. This is true, but we want to focus on the idea of slope.

   d. **Make a scale drawing of the roof on grid paper.**
      When making the scale drawing, first mark the base of 24 units horizontally. To draw the sloped line of 2/5 participants should count up two units and over five units from each endpoint of the base. The point S will be the intersection of the two lines drawn and should be directly above the midpoint of the base. Encourage participants to verify their calculated length of the altitude with the scale drawing.
   e. **Subdivide $\overline{PR}$ into four equal segments. Label each subdivision point from left to right A, B, and C. At each subdivision point draw a perpendicular to $\overline{PR}$. Label**

the intersection of each perpendicular with $\overline{PS}$ from left to right L, M and N. Find LA, MB and NC.

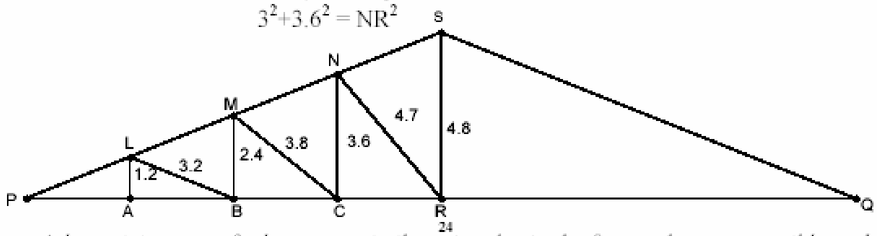
In lower grades, students might do a similar activity and find the lengths by counting squares on the grid paper and estimating. By the end of middle school, students should be able to calculate these lengths using similar triangles and the following proportions: $\frac{3}{LA} = \frac{12}{4.8}$; $\frac{6}{MB} = \frac{12}{4.8}$; $\frac{9}{NC} = \frac{12}{4.8}$.

By high school geometry, students should know that $\overline{PS}$ *is also subdivided into four equal segments since the perpendiculars are parallel to each other and they subdivide* $\overline{PR}$ *into four equal segments. With this information, students could use the Pythagorean Theorem to find LA, MB and NC. Another method uses inverse tangent to find the angle at P. Once this angle is known students can use trigonometry to find LA, MB and NC.*
Ask: What pattern do you see in the measures of LA, MB, NC, and SR? They increase at a constant rate of change, 1.2.

f. Draw segments LB, MC, and NR. Find these lengths.

Participants use the Pythagorean Theorem to find the lengths as shown below.

$3^2+1.2^2 = LB^2$
$3^2+2.4^2 = MC^2$
$3^2+3.6^2 = NR^2$



Ask participants to find as many similar triangles in the figure above as possible and justify their answers.

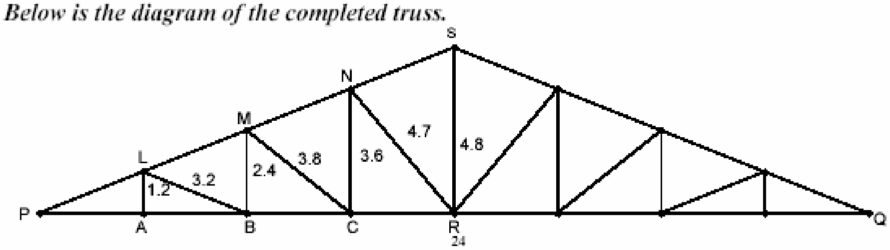
$\Delta LAP \sim \Delta MBP \sim \Delta NCP \sim \Delta SRP \sim \Delta LAB \sim \Delta SRQ$

since each triangle has a right angle and either shares angle P or has an angle with equal measure to angle P.

$\Delta PLB \sim \Delta PSQ$

since all sides are in a 1:4 ratio.

g. To complete the truss, reflect the figure across $\overline{SR}$.
Below is the diagram of the completed truss.



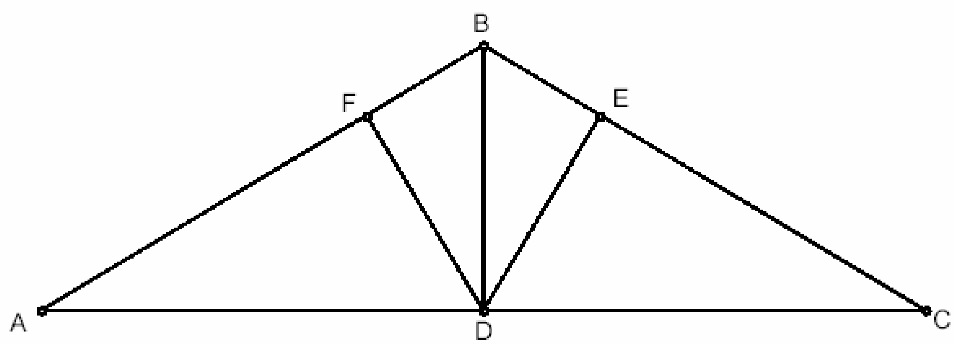
This truss can be constructed using dynamic geometry software. Then the pitch and span of the roof can be changed to demonstrate a variety of trusses.

Ask participants how the lengths found in the steps above would change if the roof span was 30 feet instead of 24 feet with the same pitch. The two trusses will be similar, so every length in the new truss will 30/24=5/4 times the length of the scale drawing.

Ask participant how the lengths would change if the pitch of the roof changes. The triangles would not be similar, so the ratios would have to be recalculated.

There are many different kinds of truss constructions. In this activity we only highlight one type of truss.

4. **Suppose a roof has a pitch of 3/5 and spans 24 ft. (AB=BC) BD is an altitude of ΔABD. DF and DE are support beams constructed perpendicular to AB and BC. Find the lengths of each support beam (DF, BD, and DE).**



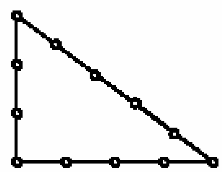
24 ft

∠BAC = $\tan^{-1}$ .6 ≈ 31.0°
AD = 1/2(24) = 12 since BD is an altitude of an isosceles triangle.
$\sin 31 = \frac{DF}{12}$; *DF = 12 sin 31 ≈ 12(.515) ≈ 6.18 ft ≈ 6 ft 2.16 in.*
DE = DF because:
∠BAD≅∠BCD since the base angles of an isosceles triangle are congruent. m∠CED=90° since DE is an altitude and m∠AFD=90° since DF is and altitude. AD=DC since the altitude cuts the base of an isosceles triangle into congruent segments. So ΔDCE ≅ ΔDAF by AAS. So, DE=DF=6 ft. 2.16 in.
$\tan 31 = \frac{BD}{12}$; *BD = 12 tan 31 ≈ 12(.6) ≈ 7.2 ft ≈ 7 ft. 2.4 in.*
Note: BD can also be found using similar triangles by drawing in the 3/5 pitch triangle.

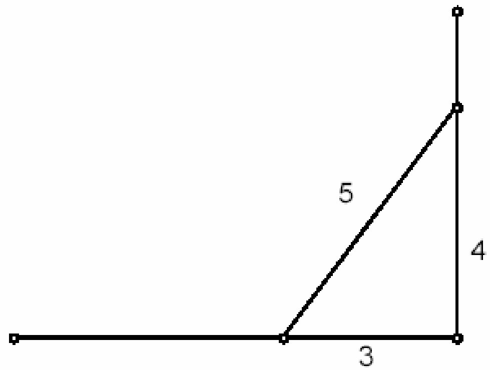
Summary: *Participants use right triangles and properties of similarity to layout a rectangular foundation and to calculate the measurements for a roofing truss.*

Activity 1: Laying Out a Foundation

As early as 2000 BC the ancient Egyptians used mathematics in very practical settings for construction. The Egyptian architects created a clever device for constructing right angles which hinges on the Pythagorean Theorem, even though Pythagoras did not actually prove his theorem until around 300 BC. The device used by the Egyptians consisted of twelve even lengths of rope tied together in a loop. This loop was pulled into a taught triangle with side lengths 3, 4 and 5 units as shown below. This would ensure a right angle between the sides lengths of 3 and 4 units.



Today, this same principle is still used in construction. When the frame for the foundation of a house is laid, workers use string to mark the sides of the frame. One side of the foundation is set between two stakes. To create a right angle, a length of three feet is marked on one side and a length of 4 feet is marked on the adjacent side. Another string of length 5 feet is held at the 3-foot mark and stretched to meet the 4-foot mark. When these two strings meet, the right angle is set. The diagram on the following page illustrates this procedure.



In this activity, you will need the following materials: 4 stakes, string, a tape measure, scissors. Your group will gather these materials, go outside to a large grass or dirt area and stake out a 10-foot by 15-foot rectangular foundation.

1. Using the supplies you have been given, stake out a 10' x 15' rectangle and use string to outline it. Use the 3-4-5 Pythagorean triple to adjust the angles at the vertices of the rectangle so that you are sure that they are 90^o.

2. Check your rectangle to be sure it is accurate. What other method can you use to ensure that the angles of your rectangle are right angles?

3. Next, we would like to add a bay window to one of the 10 foot sides of the foundation. Using the string and stakes, lay out the foundation for the bay window so that the trapezoid formed by the three walls of the bay window and the 10 foot side of the rectangle is half of a regular hexagon. Find the lengths of the three sides of the bay window and describe the method used to lay out this part of the foundation.

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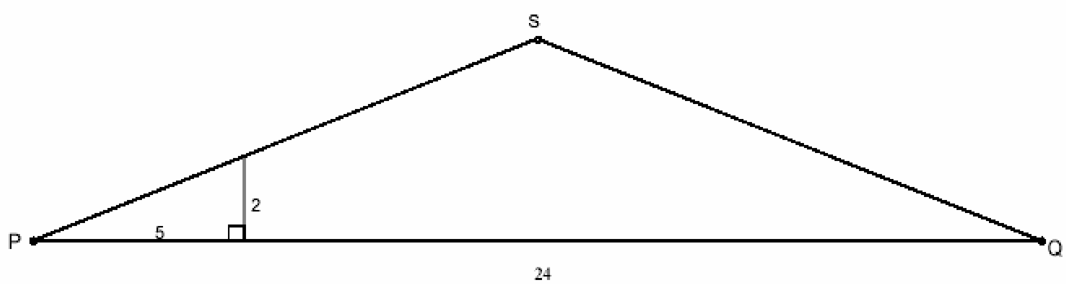

a. Calculate the measure of $\angle SPQ$.

b. Draw the altitude from point S to $\overline{PQ}$ and label the intersection R. Find SR.

c. Find PS. Express your answer in terms of feet and inches.

d. Make a scale drawing of the roof on grid paper.

e. Subdivide $\overline{PR}$ into four equal segments. Label each subdivision point from left to right A, B, and C. At each subdivision point draw a perpendicular to $\overline{PR}$. Label the intersection of each perpendicular with $\overline{PS}$ from left to right L, M and N. Find LA, MB and NC.

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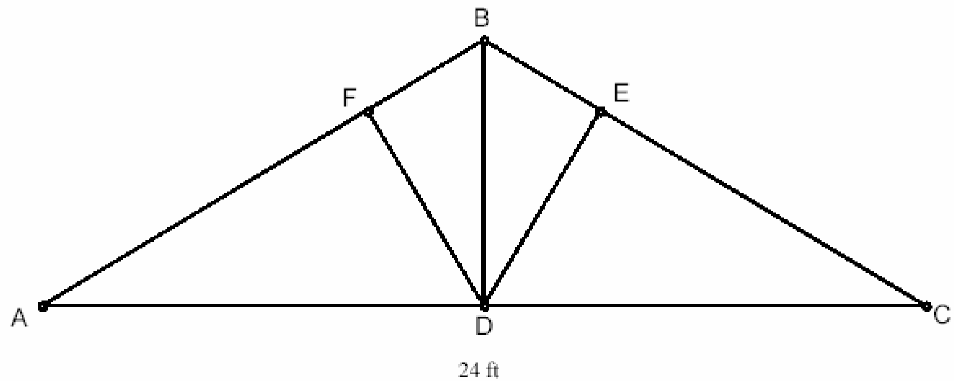
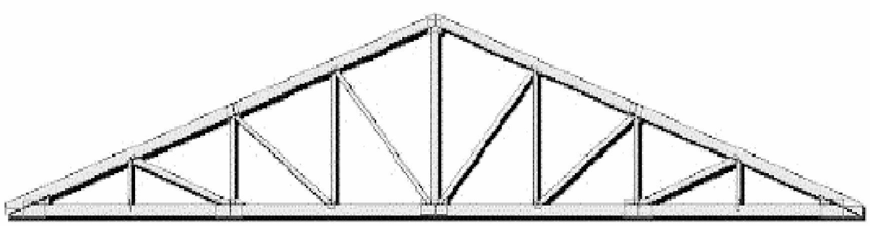


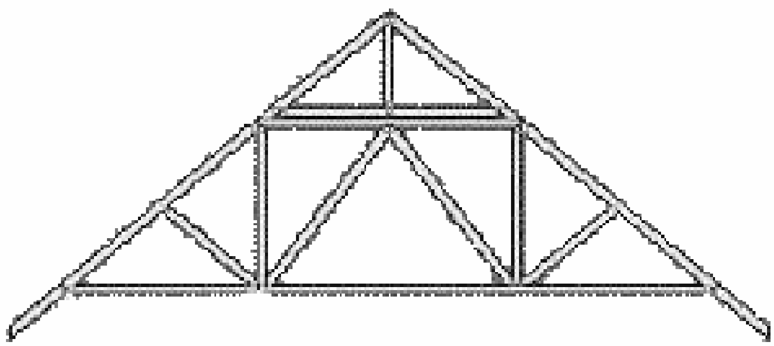
Transparency: Common Trusses

A truss is a rigid framework used in bridges and roofs. As a refinement of solid beams, trusses are an efficient means to span long distances with a minimum of materials. The following are three commonly used trusses.

Triple Howe Agricultural Truss: Depending on pitch & spacing, these trusses can clear span up to 84 feet.



2-piece "Piggyback" trusses can achieve steep pitches over large spans.



Common attic truss can provide "Bonus Room" over garage or elsewhere.