GAS GUZZLER

Focus: Investigation of the line-of-best-fit.

Objective:

Algebra I TEKS, Linear Functions #5
The student understands that linear functions can be represented in different ways and translates among their various representations.

Algebra I TEKS, Linear Functions #6
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

Terms: Independent variable, dependent variable, line-of-best-fit

Set-Up: Participants should be seated at tables in groups of 3-4.

Materials: Transparencies #18-22, Activities #21-22, graphing calculators, raw spaghetti

Prerequisites: Slope, y-intercept, scatter-plot capabilities of calculator

Procedure:

Transparency #18: Gas Guzzler Problem
Activity #21: Gas Guzzler Problem

Ask participants to describe a relation that they observe in the table of values between the amount of gas consumed and the price per gallon. Ask which relation makes more sense:

(a) as the amount of gas consumed increases, the price per gallon decreases, or
(b) as the price per gallon increases, the amount of gas consumed decreases.

In the context of this situation, (b) makes the most sense. This is how we decide which variable is the independent one and which is the dependent one. Amount of gas consumed (dependent variable) *depends* on the price per gallon (independent variable).

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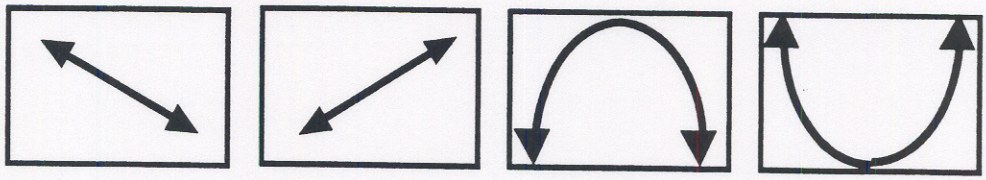
Ask participants what they would expect the "demand" curve to look like.



Ans: the first curve. (The first line is called the "demand" curve--as price increases, the demand decreases. The second line is called the "supply" curve--as price increases, the supply increases. In business, the point of intersection of the "demand" and "supply" curves is called "equilibrium" and indicates the ideal selling price. Sometimes both revenue and profit are quadratic in nature and form a curve like the third diagram.)

Emphasize Algebra I TEKS, Linear Functions #5

5.a. The student determines whether or not given situations can be represented by linear functions.

Transparency #19: Gas Guzzler Problem

Introduce the idea of "line-of-best-fit". The statistical procedure for finding the "best-fitting" straight line for a set of data points is similar, in many respects, to the procedure employed when we fit a line by eye. For instance, when we visually fit a line to a set of data, we move the ruler until we think that we have minimized the deviations of the points from the prospective line. The statistical procedure is only slightly more complicated. The procedure hunts for the "best" line that minimizes the sum of squares of the deviations from the data points and the line.

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Activity #22: Investigating Line-Of-Best-Fit
Transparency #20: Gas Guzzler Problem: Finding the Line-Of-Best-Fit

Participants are to graph the data on the coordinate grid paper, use raw spaghetti to indicate the "line-of-best-fit", and answer the questions at the bottom of the page.

Allow participants to demonstrate their approaches for determining the line-of-best-fit of their data. Discuss questions at the bottom of the activity.

Answers to Questions

1.-5. Answers will vary.

6. The x-intercept of the linear regression function is the predicted price at which no gas will be sold because the cost is so high.
The y-intercept is the predicted number of gallons of gas that could be given away if no charges were levied.

Transparency #21: Gas Guzzler Problem: Finding the Line-Of-Best-Fit

Create a Scatter Plot

Instruct participants to create a scatter plot of the points (price per gallon, amount of gas consumed) on their graphing calculator. Ask for a reasonable viewing window for the calculator, i.e. -(1, 5, 0.50, -100, 600, 50).

Note that the ways in which two variables might be related are not always shown clearly by tables of input-output data. Patterns in the data may be lost amid all the specific numbers. However, when data are displayed in a *graph*, it is often much easier to see trends and therefore to make predictions.

Make the connection between pattern seen in graph and interpretation. Note that to get from one point to another one, you must move to the right (increase in price) and then down (decrease in amount sold) or you can move to the left (decrease in price) and up (increase in amount sold).

Discuss importance of building mental images.

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Keystroke Sequences for Transparencies #21

Enter Data in Lists for Statistical Analysis

To clear existing data, press the following keys and then input new data.

STAT
4:ClrList
L1, L2, etc.
Discuss other means of clearing list data.

To input data into lists using the TI-82, press the following keys.

STAT
1:Edit
enter independent variable data in L1
enter dependent variable data in L2

Create a Scatter Plot of Data

2nd STATPLOT
1:Plot 1
ENTER
Graph: On
ENTER
Type: Scatter Plot
Xlist: (ind. var., i.e. L1)
Ylist: (dep. var., i.e. L2)
Mark: (select one)
WINDOW (select a reasonable domain and range)
GRAPH

Determine Line-of-Best-Fit

STAT
CALC
5:LinReg (ax+b)
L1, L2
ENTER

Store a Regression Equation

Once a curve of best fit has been determined on the text screen, complete the following steps.

Y=
VARS
5:Statistics
EQ
7:RegEQ

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Emphasize Algebra I TEKS, Linear Functions #5.

5.c. The student translates among algebraic, tabular, graphical, or verbal descriptions of linear functions.

Emphasize Algebra I TEKS, Linear Functions #6.

6.a. The student develops the concept of slope as rate of change and determines slopes from graphs, tables, and algebraic representations.

6.d. For situations, the student interprets the meaning of the slope and intercepts and interprets them with respect to data, symbolic representations, or graphs.

Determine Line-of-Best-Fit.

Sometimes another method of representing related variables is even more useful for prediction and forecasting purposes than tables or graphs. This 3rd method involves using *algebraic rules* that describe how output values are calculated from input values.

Questions for Discussion

- What is the equation of the line-of-best-fit? (Ans: The linear regression equation for this data is **y = -102.5x + 495**.)
- Using the regression equation and the table features of the calculator, what amount of gas consumption would be predicted for the United States when gas is prices at $0.95 per gallon? (Ans: approximately 398 gallons)
- How does the predicted amount compare to the actual amount? (Ans: predicted amount (398) is much lower than actual amount (484))
- Why do you think this discrepancy exists? (Ans: In the U. S. many more people own cars than in other countries; public transportation in other countries is often much better.)

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Transparency #22: Correlation Coefficients

Interpret Meaning of Correlation Coefficient

When the linear-regression equation is computed, the correlation coefficient "r" is automatically displayed on the calculator. The correlation coefficient measures the strength of the correlation or "the fit of the line to the data points". If "r" is close to +/- 1, there is a strong correlation among the data. The +1 indicates a direct (positive) relationship and the -1 indicates an indirect (negative) relationship.

In this problem, +1 would indicate that as the price increases the amount of gas sold would correspondingly increase. Conversely, -1 would indicate that as the prices increases, the amount of gas sold would correspondingly decrease. Therefore, we should be expecting a negative correlation.

Question for Discussion

- Ask participants what the correlation coefficient for this problem (r = .857) tells us about the data. (Ans: "r" tells us that the data is not clumped closely together and that our line-of-best-fit is only 73% ($r^2 = .857^2 = .73$) accurate at predicting consumption for any given price.
- Which data points lie the furthest from the line-of-best-fit? How do these data points affect the correlation coefficient? (Ans: The further the data points are from the line-of-best-fit, the poorer the fit and hence the lower the value of "r".)

GAS GUZZLER PROBLEM

Ramon saw a table on gas consumption in the U. S. and other countries in the San Francisco Chronicle, 1991. It seemed to him that there was a relation between the number of gallons of gas consumed and the price per gallon. What relation do you see?

GAS CONSUMPTION		
Country	Annual Consumption per capita	Price per Gallon
United States	484	0.95
Sweden	221	2.81
W. Germany	206	2.18
Norway	191	3.09
France	180	3.04
Britain	176	2.52
Austria	176	2.67
Denmark	172	3.67
Netherlands	155	3.00
Italy	148	3.90
Japan	133	3.47

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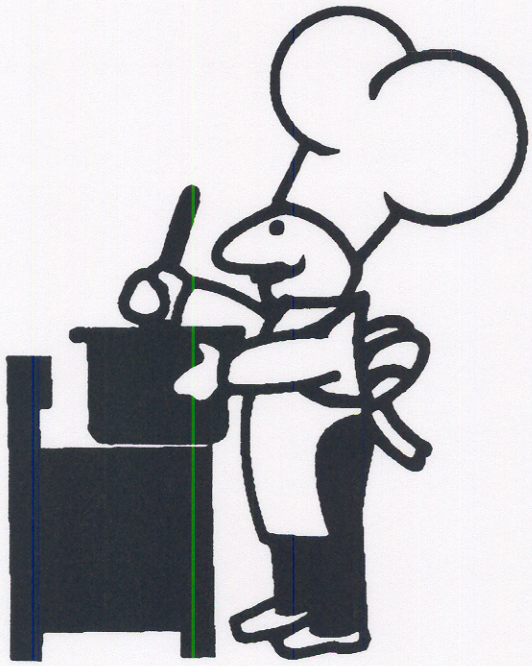
GAS GUZZLER PROBLEM

Ramon wanted to see how strong the correlation was between the price per gallon and the quantity of gas consumed. He began by graphing the data on a grid. He was working next to the kitchen where his dad was preparing spaghetti and got the bright idea of using a piece of raw spaghetti as a line. He placed the spaghetti on his grid so that there were an equal number of points on either side. This is called a "line-of-best-fit".



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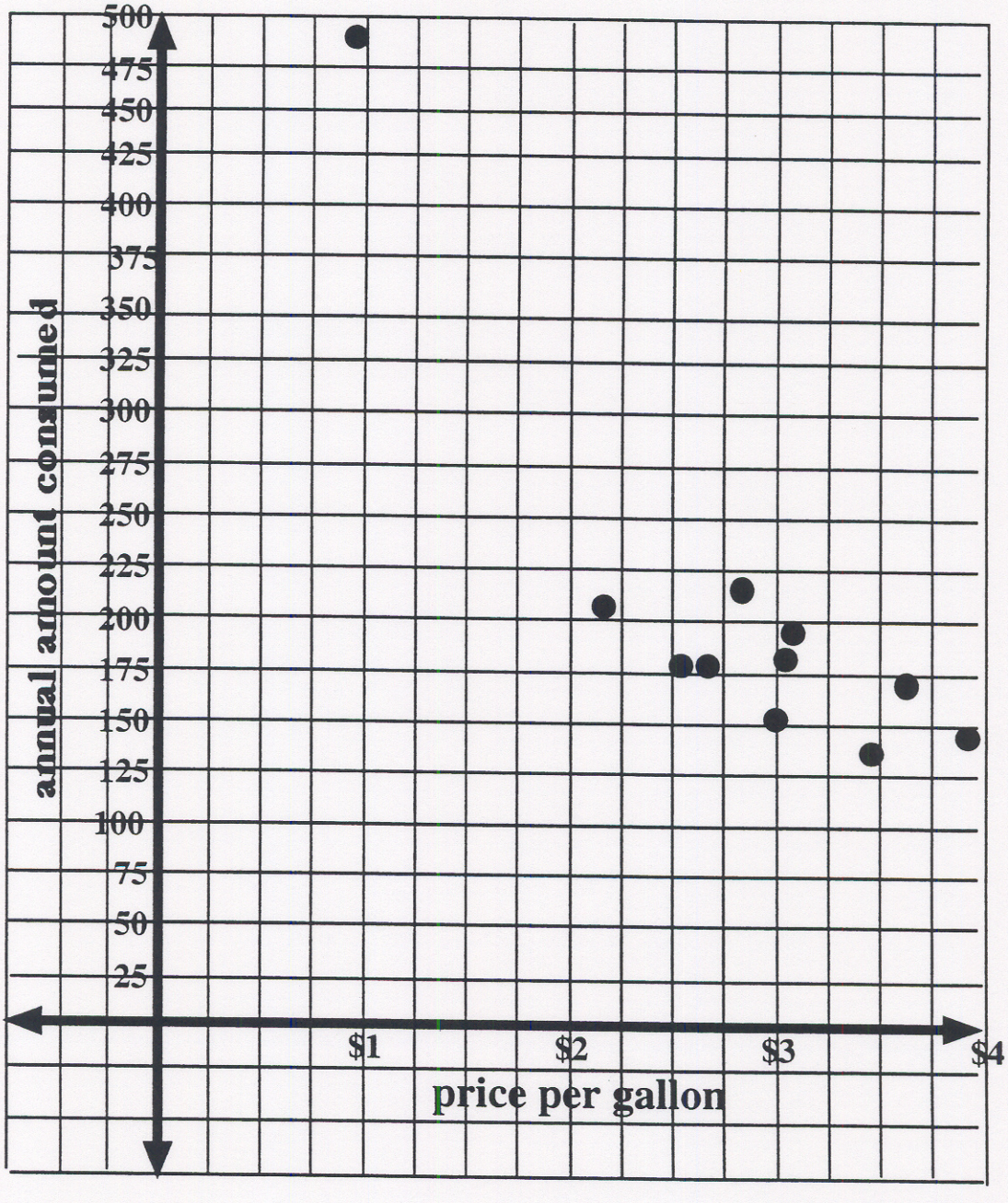
GAS GUZZLER PROBLEM: FINDING THE LINE-OF-BEST-FIT

Graph the data points on the grid and use a piece of raw spaghetti to model the "line-of-best-fit".



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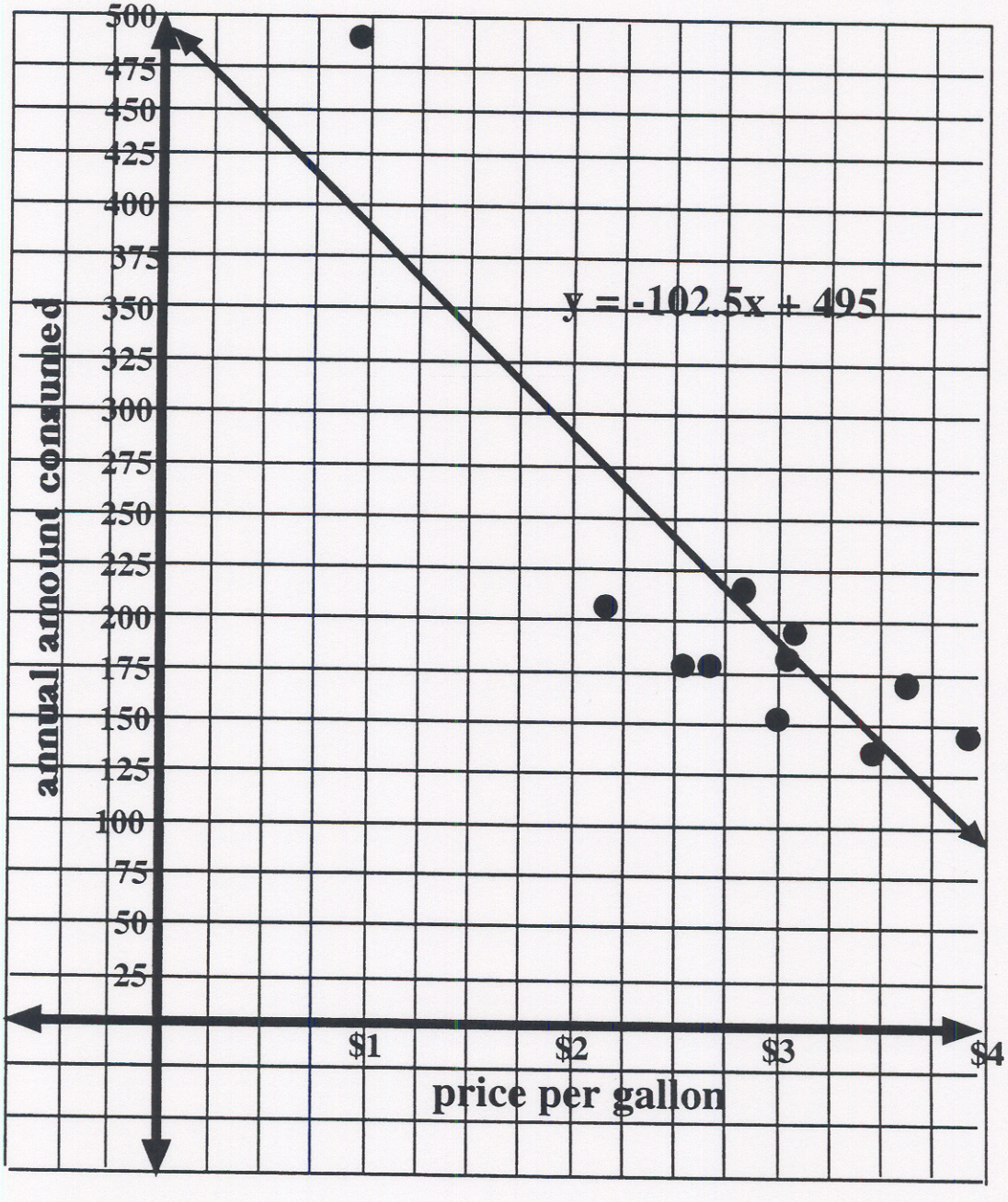

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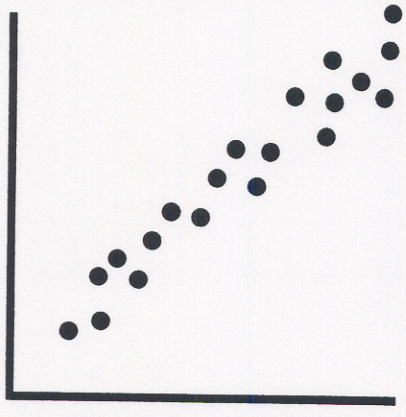
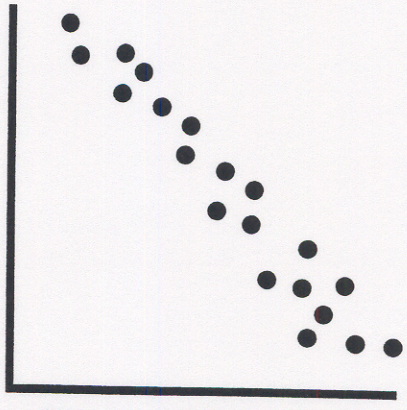
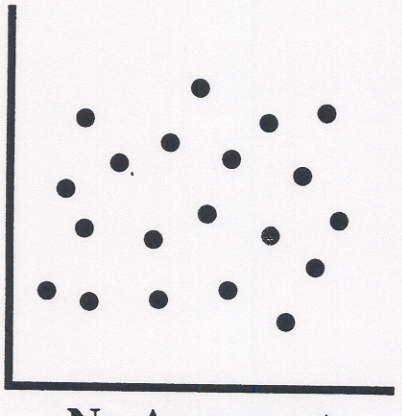
CORRELATION COEFFICIENTS



Strong Positive Linear Correlation r is near 1



Strong Negative Linear Correlation r is near -1



No Apparent Linear Correlation r is near 0

Curvilinear, but NOT Linear Correlation r is near 0

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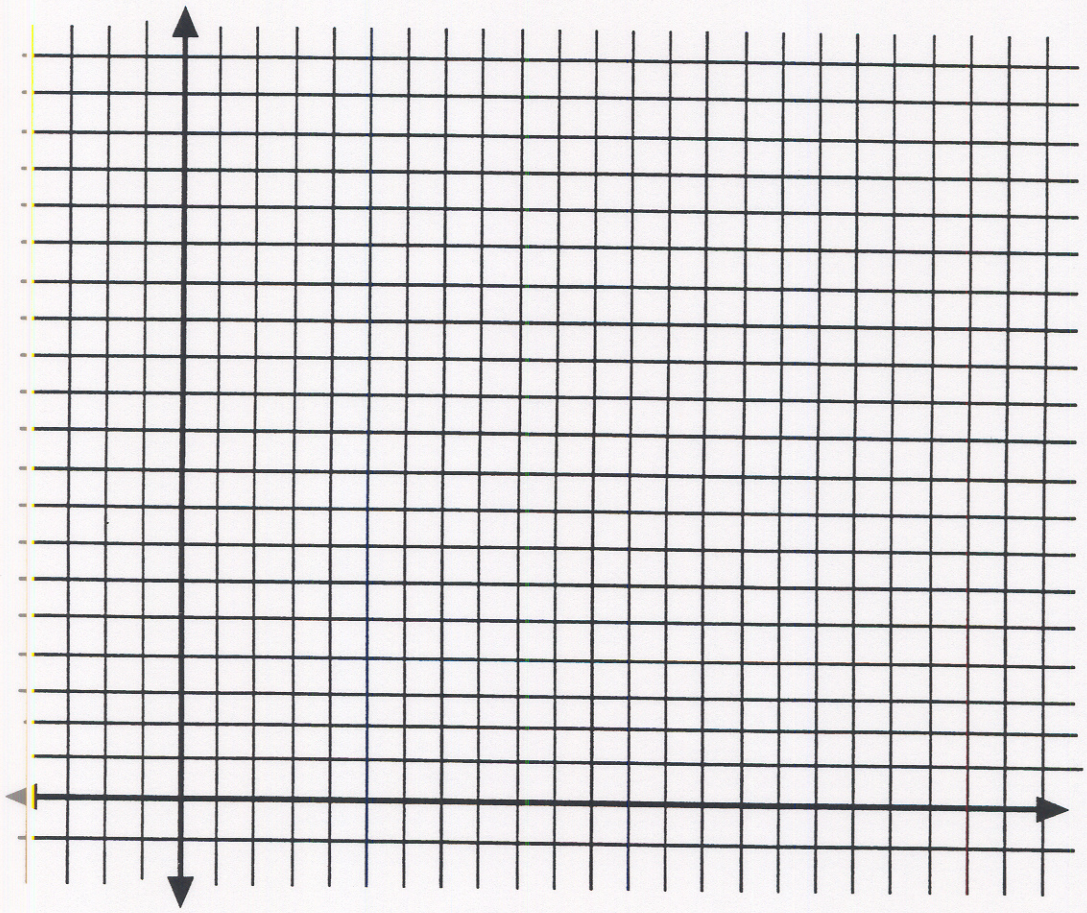
INVESTIGATING LINE-OF-BEST-FIT

Use the data from the Gas Guzzler Problem to answer the following questions.



1. Graph the data points on the coordinate grid and use a piece of raw spaghetti to model the "line-of-best-fit".
2. What are the coordinates of the points (if any) that the line contains?
3. How many points lie on either side of the line?
4. Do these points seem to "cluster" close to the line?
5. What is the slope of this line? What is the contextual meaning of the slope?
6. What is the y-intercept? the x-intercept?
   Discuss the significance of each in this problem.

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