

You're Toast, Dude!

A-E Strand(s): Algebra

Topic/Expectation

A.D.2 Rational and radical equations and functions

- a. Solve simple rational and radical equations in one variable.
- b. Graph simple rational and radical functions in two variables.

Other Topic/Expectation(s)

A.A.2 Functions

- b. Represent and interpret functions using graphs, tables, words and symbols.

Rationale

Students extend their understanding of functions to rational functions by exploring average cost. They gain experience in moving between a problem context and its mathematical model in order to solve problems and make decisions.

Instructional Task

At the You're Toast, Dude! toaster company, the weekly cost to run the factory is \$1400 and the cost of producing each toaster is an additional \$4 per toaster.

1. Write a function rule representing the weekly cost in dollars, $C(x)$, of producing x toasters.
2. What is the total cost of producing 100 toasters in one week?
3. If you produce 100 toasters in one week, what is the total production cost per toaster?
4. Will the total production cost per toaster always be the same? Justify your answer.
5. Write a function rule representing the total production cost per toaster $P(x)$ for producing x toasters.
6. Using your graphing calculator, create a graph of your function rule from question 5. Use either the graph or algebraic methods to answer the following questions:
 - a. What is the production cost per toaster if 300 toasters are produced in one week? If 500 toasters are produced in one week?
 - b. What happens to the total production cost per toaster as the number of toasters produced increases? Explain your answer.
 - c. How many toasters must be produced to have a total production cost per toaster of \$8?

Discussion/Further Questions/Extensions

Have students discuss the mathematical parameters/limitations of x , $C(x)$, and $P(x)$ compared to real-life limitations of the mathematical model for the particular situation.

Ask students to consider what happens at extremely large or small values of x , extending the mathematical discussion to asymptotes. Ask students to find the asymptotes of the production cost per toaster.

As the number of toasters, x , nears 0, the total production cost per toaster gets larger. For example, when $x = 10$ toasters, the total production cost per toaster is \$144. When $x = 5$, the total production cost per toaster is \$284. When $x = 0$ the function cannot be solved for $P(x)$.

As the number of toasters, x , becomes larger and larger, the average cost per toaster nears \$4. For example, when $x = 5,000$ toasters, the average cost per toaster is \$4.28. When $P(x) = 4$ the function cannot be solved for x . The horizontal asymptote of the function is $y = 4$, and the vertical asymptote is $x = 0$.

Sample Solutions

1. Write a function rule representing the weekly cost in dollars, $C(x)$, of producing x toasters.

$$C(x) = 4x + 1400$$

2. What is the total cost of producing 100 toasters in one week?

$$C(100) = 4(100) + 1400 = 1800$$

It will cost \$1800 to produce 100 toasters in one week.

3. If you produce 100 toasters in one week, what is the total production cost per toaster?

$$1800 / 100 = 18. \text{ If 100 toasters are produced the total production cost per toaster is } \$18.$$

4. Will the total production cost per toaster always be the same? Justify your answer.

No. Justifications may vary. If 200 toasters are produced in one week, the cost is $C(200) = 4(200) + 1400 = 2200$. The total production cost is $\$2200 / 200 = \11 per toaster. Since \$11 does not equal \$18, the total production cost per toaster is not the same when the number of toasters produced varies.

5. Write a function rule representing the total production cost per toaster $P(x)$ for producing x toasters.

$$P(x) = (4x + 1400) / x \text{ or } P(x) = 4 + (1400/x) \text{ or } P(x) = C(x)/x$$

6. Using your graphing calculator, create a graph of your function rule from question 5. Use either the graph or algebraic methods to answer the following questions:

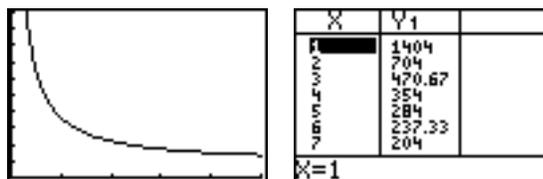
- a. What is the production cost per toaster if 300 toasters are produced in one week? If 500 toasters are produced in one week?

If 300 toasters are produced, the total production cost per toaster is \$8.67.

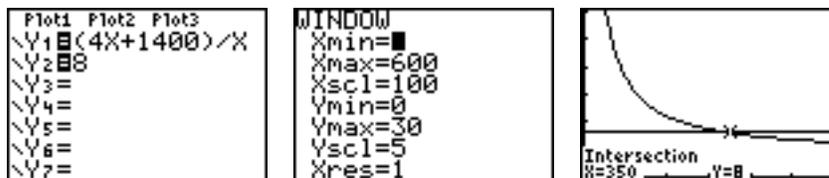
If 500 toasters are produced, the total production cost per toaster is \$6.80.

- b. What happens to the total production cost per toaster as the number of toasters produced increases? Explain your answer.

As the number of toasters produced increases, the total production cost per toaster decreases. Looking at the graph will show that as x increases, y decreases. In addition, looking at the table of values in the graphing calculator you can see that as the x values increase, the y values decrease.



- c. How many toasters must be produced to have a total production cost per toaster of \$8?



The viewing window above shows that the intersection of $y = 4 + (1400/x)$ and $y = 8$ is the point (350, 8). Therefore, when 350 toasters are produced in one week, the total production cost per toaster is \$8.